

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS (SYLLABUS D)

4024/21
Paper 2
May/June 2010
2 hours 30 minutes
Additional Materials: Answer Booklet/Paper
Graph paper (2 sheets)
Electronic calculator Geometrical instruments

Mathematical tables (optional)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

## Section A

Answer all questions.

## Section B

Answer any four questions.
Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100 .

## Section A [52 marks]

Answer all questions in this section

1 A function is defined by $\mathrm{f}(x)=\frac{x-2}{5}$.
(a) Find $\mathrm{f}(7)$.
(b) Given that $\mathrm{f}(t)=t$, find $t$.
(c) Find $\mathrm{f}^{-1}(x)$.

2 Wasim owns a shop.
The table shows the cost price and selling price of three items in his shop.

| Item | Cost Price (\$) | Selling Price (\$) |
| :---: | :---: | :---: |
| Trampoline | 48 | 66 |
| Swing | $x$ | 19.50 |
| Bicycle | 82 | 110 |

(a) Calculate his percentage profit when he sells a trampoline.
(b) Wasim makes a profit of $30 \%$ when he sells a swing.

Calculate the cost price, $\$ x$, of a swing.
(c) In a sale, the selling price of a bicycle is reduced by $20 \%$.
(i) Find the sale price of a bicycle.
(ii)

## THURSDAY SPECIAL

Save $10 \%$ off the sale price.

Jaspreet bought a bicycle on a Thursday.
Calculate the difference between the amount Jaspreet paid and the cost price.
$\qquad$

3 The diagram below is a sketch of a rectangular field WXYZ. $W X=65 \mathrm{~m}$ and $X Y=40 \mathrm{~m}$.

(a) Using a scale of 1 cm to 5 m , make a scale drawing of the field.
(b) (i) On your scale drawing, construct the locus of all the points in the field that are

I equidistant from $Z$ and $Y$,
II 45 m from $X$.
(ii) Hence label the region, $\mathbf{R}$, of points that are nearer to $Z$ than $Y$ and not more than 45 m from $X$.
(c) A post $P$ is placed in the region $\mathbf{R}$ so that it is at the maximum distance from $Y$.

A post $Q$ is placed at the midpoint of the line $Z Y$.
(i) Label the posts $P$ and $Q$ on your drawing.
(ii) By measurement, find
(a) the actual distance between the posts $P$ and $Q$ in the field,
(b) the obtuse angle $P Q$ makes with $Q Y$.

4 (a) Express as a single fraction in its simplest form $\frac{4}{x+3}-\frac{3}{2 x-1}$.
(b) It is given that $k=\sqrt{2 l m+3 n}$.

Express $m$ in terms of $k, l$ and $n$.
(c) Solve the equation $3 x^{2}-4 x-16=0$.

Give your answers correct to 2 decimal places.

5 (a) Ben travels to work by bus, by bicycle or by car.
The probability that he travels by bus on any day is 0.5 .
The probability that he travels by bicycle on any day is 0.2 .
The tree diagram below shows some of the probabilities of the possible journeys on Monday and Tuesday.

(i) Find the values of $p, q$ and $r$.
(ii) Find the probability that Ben travels to work
(a) by bus on both Monday and Tuesday,
(b) by bus on one day and by bicycle on the other day.
(b) The Venn diagram shows the three means of transport used by a group of workers during a week.

(i) How many used both a bus and a car but not a bicycle?
(ii) Twice as many only used a bicycle as only used a bus.

There were 78 workers in the group.
How many used a bus only?

$A, B$ and $C$ are points on the circumference of a circle, centre $O$.
$A E$ and $C E$ are tangents to the circle.
$A \hat{C} E=68^{\circ}$ and $\hat{B C O}=46^{\circ}$.
(a) Calculate
(i) $A \hat{O} C$,
(ii) $A \hat{E} C$.
(b) Find the three angles of the triangle $A B C$ and hence state the name given to this special triangle.

## 7 Answer the WHOLE of this question on a sheet of graph paper.

The table below shows the amount of time spent playing sport each week by 80 students.

| Time ( $t$ hours) | $0<t \leqslant 2$ | $2<t \leqslant 4$ | $4<t \leqslant 6$ | $6<t \leqslant 8$ | $8<t \leqslant 10$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 20 | 31 | 22 | 5 | 2 |

(a) Calculate an estimate of the mean time spent playing sport each week by the students.
(b) Copy and complete the cumulative frequency table below.

| Time ( $t$ hours $)$ | $t \leqslant 2$ | $t \leqslant 4$ | $t \leqslant 6$ | $t \leqslant 8$ | $t \leqslant 10$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Cumulative <br> frequency | 20 | 51 |  |  | 80 |

(c) Using a horizontal scale of 1 cm to represent 1 hour and a vertical scale of 1 cm to represent 5 students, draw a smooth cumulative frequency curve for this data.
(d) Use your graph to estimate
(i) the median,
(ii) the interquartile range.

6

## Section B [48 marks]

Answer four questions in this section.
Each question in this section carries 12 marks.

## 8 Answer the WHOLE of this question on a sheet of graph paper.

The table below shows some values of $x$ and the corresponding values of $y$ for

$$
y=\frac{12}{x}-x \text {. }
$$

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 11 | 4 | 1 | -1 | $p$ | -4 |

(a) Calculate $p$.
(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal $x$-axis for $0 \leqslant x \leqslant 6$.

Using a scale of 1 cm to represent 1 unit, draw a vertical $y$-axis for $-4 \leqslant y \leqslant 14$.
On your axes, plot the points given in the table and join them with a smooth curve.
(c) Use your graph to solve the equation $\frac{12}{x}-x=2$ in the range $1 \leqslant x \leqslant 6$.
(d) The equation $\frac{12}{x}=2 x$ can be solved using the intersection of your curve and a straight line.
(i) State the equation of this straight line.
(ii) By drawing this straight line, solve the equation $\frac{12}{x}=2 x$.
(e) The points $A$ and $B$ are $(1,11)$ and $(4,-1)$ respectively.

Find the gradient of the line $A B$.
(f) The line $l$ is parallel to $A B$ and is a tangent to the curve $y=\frac{12}{x}-x$.
(i) Draw the line $l$.
(ii) Find the coordinates of the point where $l$ crosses the $y$-axis.
(iii) Hence find the equation of the line $l$.


Diagram I
Diagram I shows a vessel in the shape of a prism.
The cross-section $O A B$ is a sector of a circle of radius 8 cm and $A \hat{O} B=90^{\circ}$.
(a) (i) Calculate the perimeter of the sector $O A B$.
(ii) The vessel, which stands on a horizontal table, contains $800 \mathrm{~cm}^{3}$ of water, shown shaded in the diagram.

Calculate the depth of the water in the vessel.
(b)


Diagram II


Diagram III

The vessel is now placed so that its curved surface is in contact with the horizontal table as shown in Diagram II.

Diagram III shows the cross-section of the vessel with the shaded section representing the water. $P$ is the midpoint of the arc $A B$ and is in contact with the table.
$Q$ is the point on the water surface, $M N$, which is vertically above $P$.
It is given that $O Q=x$ centimetres.
(i) (a) Write down an expression, in terms of $x$, for $M N$.
(b) Show that the shaded area in Diagram III is $\left(16 \pi-x^{2}\right)$ square centimetres.
(ii) The vessel still contains $800 \mathrm{~cm}^{3}$ of water.

Given that the length of the vessel is 20 cm , find the value of $x$.

$A B C D E F G H$ is an octagon with exactly two lines of symmetry.
These are shown by the dashed lines $P Q$ and $R S$ which intersect at $O$.
$A \hat{B} C=140^{\circ}$.
(a) Find
(i) $E \hat{F} G$,
(ii) $F \hat{G} H$.
(b) The diagram shows part of the octagon.
$T$ is the point such that $P T$ and $T S$ are perpendicular. $P B=50 \mathrm{~cm}, B T=23 \mathrm{~cm}$ and $C S=20 \mathrm{~cm}$.

(i) Calculate $C T$.
(ii) Calculate the area of the pentagon $P B C S O$.
(iii) Hence find the area of the octagon $A B C D E F G H$.
(iv)


The octagonal shape $A B C D E F G H$ is to be cut from a rectangular piece of card where the length and breadth, measured in centimetres, are integers.
The remaining card is wasted.
The card that is wasted must be kept to a minimum.
Find
(a) the length and breadth of the rectangular piece of card,
(b) the area of the card that is wasted.

11 (a) The diagram shows triangles $A, B, C$ and $D$.

(i) Triangle $A$ is mapped onto triangle $B$ by the translation $\mathbf{T}$.

Write down the column vector that represents $\mathbf{T}$.
(ii) Describe fully the single transformation that maps triangle $A$ onto triangle $C$.
(iii) Triangle $A$ is mapped onto triangle $D$ by a single transformation.

What is the name of this transformation?
(iv) Triangle $D$ has one line of symmetry.

Find the equation of this line of symmetry.
(b) $K$ is the point $(p, q)$.
(i) The transformation $\mathbf{U}$ is represented by the matrix $\left(\begin{array}{rr}0 & -1 \\ -1 & 0\end{array}\right)$.

This transformation maps the point $K$ onto the point $L$.
Find, in terms of $p$ and $q$, the coordinates of $L$.
(ii) The transformation $\mathbf{V}$ is a rotation $90^{\circ}$ clockwise about the point $(0,0)$.

This transformation maps the point $K$ onto the point $M$.
Find, in terms of $p$ and $q$, the coordinates of $M$.
(iii) The point $L$ is mapped onto the point $M$ by the single transformation $\mathbf{W}$.

Find the matrix that represents the transformation $\mathbf{W}$.

12 (a)


In the diagram, $O P Q R$ is a parallelogram.
$\overrightarrow{O P}=\mathbf{p}$ and $\overrightarrow{O Q}=\mathbf{q}$.
$S$ is the point on $O P$ such that $O S: S P$ is $1: 3$.
$T$ is the midpoint of $O R$.
Giving your answers in their simplest form, find, in terms of $\mathbf{p}$ and $\mathbf{q}$,
(i) $\overrightarrow{Q P}$,
(ii) $\overrightarrow{T S}$.
(b) In triangle $W X Y, W X=24 \mathrm{~cm}, W Y=17 \mathrm{~cm}$ and $X \hat{W} Y=55^{\circ}$.


Diagram I
(i) Calculate
(a) the area of triangle $W X Y$,
(b) $X Y$.
(ii) [Volume of a pyramid $=\frac{1}{3} \times$ base area $\times$ height]

The triangle $W X Y$ shown in Diagram I forms the horizontal base of the triangular pyramid $V W X Y$, shown in Diagram II.

The vertex $V$ is vertically above $Z$, a point on $W X$. $W V=15 \mathrm{~cm}$ and $W Z=\frac{1}{4} W X$.


Diagram II
(a) Calculate $V Z$.
(b) Hence find the volume of the pyramid.

BLANK PAGE

## BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

